

Analysis of Reinforced Concrete Beams Strengthened with Composites

Subjected to Fatigue Loading

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Synopsis: Use of high strength composites for repair and rehabilitation of bridges and parking decks is steadily increasing. Since these structures are subjected to fatigue loading, the performance of strengthened beams under this type of loading needs to be evaluated. An analytical procedure that incorporates cyclic creep of concrete and degradation of flexural stiffness is presented. The method is verified by computing cycle dependent deflections and comparing them with experimental results. The results presented in this paper also provide a summary of an experimental investigation, in which reinforced concrete beams were strengthened with glass fabrics (sheets) and subjected to fatigue loading. The comparison shows that the analytical model provides reasonably accurate prediction of deflections, for both reinforced beams and reinforced concrete beams reinforced with composites. Although Glass fiber composites were used for the evaluation, the model is also applicable to other types of fibers.

Keywords

fatigue; analysis; fatigue; strength; stiffness; deflection; reinforced concrete; carbon; glass; fiber reinforced polymer composite; strengthening

Biographical Sketches

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Introduction

There is an increasing interest in the use of high strength composites for repair and rehabilitation of reinforced concrete elements. Since most of these elements are structural members of bridges or parking garages, there is a need to understand the behavior of strengthened elements under repeated loading. A small number of researchers have investigated the fatigue properties of reinforced concrete beams strengthened with glass and carbon fibers¹⁻⁴.

Meier reported two fatigue tests of reinforced concrete beams strengthened with a glass/carbon-hybrid sheet¹. Barnes and Mays reported fatigue test results of two reinforced concrete beams and three reinforced concrete beams strengthened with CFRP plates². Papakonstantinou also reported on fatigue tests of six reinforced concrete beams and eight reinforced concrete beams strengthened with GFRP plates⁴. The results of all these tests indicate that beams failed in a primary flexural mode and initial failure was due to the steel fracture rather than failure of concrete, adhesive or fracture of the FRP plates. There were no distinguishable differences in behavior between the strengthened and non-strengthened beams during load cycling. The number of cycles to failure varied according to the stress range of steel reinforcement.

Shahawy and Beitelman tested six tee-beams under fatigue loading³, using a load range of 0-25% of the ultimate load. Two beams were used as control beams, two were fully wrapped with two layers of CFRP and two were wrapped with three layers of CFRP on the tension side. From the results of these tests it was concluded that the failure was based on the fracture of steel, the use of FRP increases the fatigue life of the beams as well as that the increase of the number of layers of CFRP results in a further increase of the fatigue life of the beams.

The aforementioned investigations were conducted mainly to obtain the fatigue strength of the strengthened beams. There is no study so far to address the serviceability aspect of the strengthened beams subjected to fatigue loading. Since concrete beams subjected to a repeated loading will experience an increase in deflections, satisfactory prediction of this time dependent quantity may be important in cases where serviceability criteria govern the design. In addition excessive deflection increase could signal the impending failure. In certain cases there could be change in stress in reinforcement due to creeping of concrete.

In this paper an analytical model is presented which can be used to predict the increase in deflection of reinforced concrete beams strengthened with high strength fiber reinforced composite fabric. The analytical results are compared with available experimental data. The model can also be used to compute any changes in stresses due to cyclic creep of concrete.

Analytical Procedure

Balaguru and Shah reported an analytical model, which can be used to predict the increase of deflections of reinforced concrete beams, subjected to fatigue loading⁵. Based on this model, the two major contributing factors are: (i) cyclic creep of concrete and (ii) degradation of flexural stiffness due to increase in cracking and reduction in modulus of rupture under fatigue loading.

The cyclic creep strain of concrete can be expressed as the sum of two strain components; a mean strain component, based on $\sigma_m = ((\sigma_{max} + \sigma_{min})/2)/f_c'$ and a cyclic strain component, based on $\Delta = (\sigma_{max} - \sigma_{min})/f_c'$. A regression equation based on experimental results was obtained as:

$$\epsilon_c = 129 \cdot \sigma_m \cdot t^{1/3} + 17.8 \cdot \sigma_m \cdot \Delta \cdot N^{1/3} \quad (1)$$

where ϵ_c is the cyclic creep strain in micro mm/mm

Δ is the stress range expressed as a fraction of the compressive strength

σ_m is the mean stress expressed as a fraction of the compressive strength

σ_{max} is the maximum applied compressive stress in concrete

σ_{\min} is the minimum applied compressive stress in concrete
 N is the number of cycles
 t is the time from start of loading in hours.

Once the cyclic creep strain is known, the cycle-dependent secant modulus for concrete in compression, E_N can be expressed using the equation:

$$E_N = \frac{\sigma_{\max}}{\frac{\sigma_{\max}}{E} + \epsilon_c} \quad (2)$$

where E is the initial secant modulus

E_N is the cyclic modulus after N number of cycles

Use of this cycle dependent modulus for the analysis is presented in the following section.

Outline of Analytical Procedure for a Rectangular Beam

The analytical procedure is explained using a rectangular section reinforced with steel and strengthened with composites. The details of beam cross section, strain and stress distribution at working loads are shown in Fig. 1.

The depth of the neutral axis, kd can be computed using the force equilibrium equation. The contribution of composite is considered to be similar to the contribution of steel reinforcement. The area of composite is multiplied by the modular ration, n_f . In addition to the assumptions of classical bending theory, it is assumed that: (i) there is a perfect bond between the composite plate and the beam up to failure and (ii) the behavior of the composite plate is linearly elastic up to failure.

$$b \frac{(kd)^2}{2} + (n-1)A_s'(kd-d') = nA_s(d-kd) + n_f A_f(h-kd) \quad (3)$$

where A_s is the cross sectional area of the reinforcing bars in tension side
 A_f is the cross sectional area of the composite plate in tension face
 A_s' is the cross sectional area of the reinforcing bars in compression side
 b is the width of the beam
 d is the depth of the reinforcing bars in the tension side
 d' is the depth of the reinforcing bars in the compression side
 kd is the depth of the neutral axis
 h is the thickness of the beam
 n is the modular ratio of steel reinforcement given by the equation:

$$n = \frac{E_s}{E \text{ or } E_N}$$

E_s is the modulus of elasticity of the reinforcing bars

n_f is the modular ratio of the composite given by the equation:

$$n_f = \frac{E_f}{E \text{ or } E_N}$$

E_f is the modulus of elasticity of the composite plate

Once the depth of the neutral axis is known, the moment of inertia of cracked section, I_{cr} can be computed using the following equation:

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d-kd)^2 + n_f A_f(h-kd)^2 + (n-1)A_s'(kd-d')^2 \quad (4)$$

For the uncracked section, the gross moment of inertia, I_g is given by:

$$I_g = \frac{bh^3}{12} \quad (5)$$

The stresses at the extreme compression fiber, σ_c , the composite in the tension fiber, σ_f , as well as at the reinforcement in the tension, σ_s , and compression side, σ_s' , can be calculated using the I_{cr} and the principles of mechanics:

$$\sigma_c = \frac{M}{I_{cr}} kd \quad (6)$$

$$\sigma_f = \frac{M}{I_{cr}} (h - kd) n_f \quad (7)$$

$$\sigma_s = \frac{M}{I_{cr}} (d - kd) n \quad (8)$$

$$\sigma_s' = \frac{M}{I_{cr}} (kd - d') n \quad (9)$$

where M is the applied moment.

For the first cycle; maximum stress in concrete:

$$\sigma_{\max} = \frac{M_{\max}}{I_{cr}} kd \quad (10)$$

while minimum stress in concrete:

$$\sigma_{\min} = \frac{M_{\min}}{I_{cr}} kd \quad (11)$$

where

M_{\max} is the maximum applied moment

M_{\min} is the minimum applied moment

Once σ_{\max} and σ_{\min} in concrete are known, the cycle dependent modulus E_N for a given number of cycles can be calculated using Eq. 1. and 2.

Computation of Flexural Stiffness Degradation

In the case of static loading, the contribution of concrete in the tension zone is accounted for, by using the effective moment of inertia. The effective moment of inertia can be obtained using the formula⁶:

$$I_e = I_{cr} + \left(\frac{M_{cr}}{M_a} \right)^3 (I_g - I_{cr}) \quad (12)$$

where I_e , I_{cr} , I_g are the effective, cracked and gross moment of inertia, M_a is the maximum value of the applied moment along the beam, and M_{cr} is the cracking moment given by $M_{cr} = I_g f_r / (h - \bar{y})$ where f_r is the modulus of rupture of concrete and \bar{y} is the depth of neutral axis of the uncracked section.

The stiffness of the beam will reduce when subjected to fatigue loading due to the fatigue of concrete in the tension zone. This progressive reduction in the stiffness can be accounted for by using a reduced cycle-dependent modulus of rupture. This relationship can be expressed as:

$$f_{r,N} = f_r \cdot \left(1 - \frac{\log_{10} N}{10.954} \right) \quad (13)$$

It should be noted that the increase of the number of cycles will lead to the reduction of the modulus of rupture, cracking moment M_{cr} and consequently the effective moment of inertia I_e .

For the static loads, the deflections are calculated using the initial effective moment of inertia (I_e) and the first cycle modulus of elasticity of concrete (E), that is:

$$\delta = \frac{f(\text{load}, \text{span})}{E \cdot I_e} \quad (14)$$

where δ is the instantaneous deflection

$f(\text{load}, \text{span})$ is a function for the particular load and span arrangement

and EI_e is the initial effective stiffness of the beam

In the case of fatigue loading equation (14) changes to:

$$\delta = \frac{f(\text{load}, \text{span})}{E_N \cdot I_{e,N}} \quad (15)$$

where $E_N I_{e,N}$ is the reduced stiffness of the beam after N cycles.

E_N is given by equation (2) and $I_{e,N}$ is given by the following equation:

$$I_{e,N} = I_{cr,N} + \left(\frac{M_{cr,N}}{M_a} \right)^3 (I_g - I_{cr,N}) \quad (16)$$

$$M_{cr,N} = \frac{I_g}{(h - \bar{y})} f_{r,N} \quad (17)$$

where $f_{r,N}$ is given by equation (13) and

$I_{cr,N}$ is the moment of inertia calculated using E_N

The tensile stiffness degradation is accounted by the term $M_{cr,N}$ while the effect of the concrete creep is included in the terms E_N and $I_{cr,N}$.

A step by step procedure and a numerical example are presented in the Appendix.

Details of Experimental Results Used for Evaluation of the Model

The experiments reported by Papakonstantinou⁴ were used for evaluating the model. In this investigation seventeen reinforced concrete beams, nine of them strengthened with a Glass Fiber Reinforced Polymer (GFRP) system were tested in third-point flexure. Four beams were initially tested under static load in order to establish their ultimate load carrying capacity.

The average compressive strength of concrete was 40 MPa whereas the yield strength of the steel was 427 MPa. The beams were 1320 mm long, 152 mm wide and 152 mm thick and were simply supported over a span of 1220 mm. Two 12.7 mm diameter bars were used as flexural tension reinforcement, and two more were placed in the compression region for fabrication ease. For shear reinforcement, 9.5 mm bars were used. The flexural and shear reinforcement was designed to ensure flexural failure. Twenty 9.5 mm stirrups were placed in the beam as shear reinforcement at a spacing of 50 mm in the two outer thirds of the beam, and two stirrups were spaced evenly in the center third, zero-shear/constant-moment portion of the beam. There was a 25 mm clear cover of concrete for each side of the stirrups except for the top surface, which had a 13 mm cover. Specimen details are shown in Fig. 2.

Two designations: N, for non-repaired reinforced concrete beams, and S, for beams strengthened with GFRP composite sheets, were used. Each beam was numbered in a consecutive order. All the repaired beams were reinforced over their entire tensile face with a GFRP sheet. The GFRP system had a strength of 1,730 MPa and a stiffness of 72.4 GPa. Repaired specimens were allowed to cure at least 10 days after the FRP application.

A summary of the beams and the loading conditions are presented in Table 1. The maximum and minimum loads applied are presented for the fatigue tests. It should be noted that the maximum loads were always determined based on the strain on the steel reinforcement. The percentage of ultimate capacity is determined from the static tests. The minimum applied load was necessary to avoid the instability because the supports provide restrain in only one direction.

The tests were carried out using a 270 kN capacity hydraulic actuator. The actuator was operated under load control for the cycling loading and under stroke displacement control for the static tests. The load was applied at the third points of the beam within the 1220 mm supported span. All contact points along the beam were supported with rollers to ensure that the load was applied evenly during testing (Fig. 2).

Strains on steel, concrete and the GFRP sheet as well as deflections at midspan were recorded during testing in order to study their change under static and fatigue loading conditions. Two electrical resistance strain gages were attached on opposite sides at the mid-span of each longitudinal reinforcing bar to measure the axial strain in the bar. Strain gages were also located on the top concrete surface as well as on the GFRP sheet at selected locations (Fig. 2).

The deflection at mid-span was measured using a “yoke” deflection measuring system. This system, illustrated in Fig. 2, consists of an aluminum frame, bolted on the neutral axis of the beam at the end supports and two LVDT’s mounted on the “yoke” at midspan, one on each side of the beam. The LVDT’s measure the deflection of the specimen through contact with two aluminum plates, bonded to the beam with epoxy. The average of the two measurements represents the net mid-span deflection.

Experimental Results from Static Tests

Two non-repaired (N-1 and N-2) and one repaired (S-4) beams were tested under monotonic loading conditions and were used as control beams for the fatigue testing. The experimental results are shown in Fig. 3. The strengthened beam was stiffer than the control beams before and after reinforcement yielding. FRP increased the ultimate load capacity by about 50% and the reduction in ductility was less than 10%. The failure was initiated by yielding of the reinforcement and subsequent delamination of the GFRP sheets.

Experimental Results From Fatigue Tests

Fourteen beams were tested under cycling load at different load ranges. Six of them were non-repaired and eight repaired. The maximum load in each case was chosen based on the maximum stress on the steel reinforcement. The minimum load depended on the frequency of the loading and was selected to ensure stability of the test set up. The maximum number of cycles applied was limited to 2×10^6 cycles. During all fatigue tests, static tests were conducted on a regular basis in order to monitor the damage accumulation and its effect on the deflection and stress values. All tests were conducted at 2 or 3 Hz (Table 1).

The data in the Table 1 include the maximum and minimum applied load and strain on the reinforcing bars and on the concrete (compression), as well as the ratio of applied load to ultimate (P_{\max}/P_{ult}) and yielding loads (P_{\max}/P_y), the number of cycles to failure. The ultimate values were determined from the original static control tests.

The change of deflections with the increase of the number of cycles for all beams that failed under cycling loading are presented in Fig. 4. For each beam the deflection was recorded during monotonic loading tests. Note that in all beams there was an initial substantial increase of the deflections, followed by a stable region where the deflection increases were minimal. Just before failure there was another period of substantial deflection increase. It can also be observed that, in all beams that failed after 400,000 cycles, there is a more gradual increase in deflections with increase in number of cycles. This may be caused by a greater number of cracks with increased cycling. Generally, however, the deflection remained the same until approximately 100,000 cycles before failure occurred. This is important, since one can detect an upcoming failure by monitoring the deflections.

Comparison of Analytical and Experimental Results

Comparison of experimental and analytical deflection values are presented in Figs. 5 to 18. These figures show graphs of normalized deflection vs. number of cycles for each beam subjected to fatigue loading. The values of the experimental deflections were normalized using the first value recorded after the beginning of the experiment. The normalized theoretical values were obtained by dividing each of the theoretical values by the value calculated for the corresponding experimental value.

Fig. 5. presents the comparison data for the beam N-3. It is evident that there is a very good correlation between the predicted theoretical and experimental values. The predicted values are also close to the experimental for the beams N-6, N-7 and N-8 (Figs. 8, 9 and 10). There is only some inconsistency between the analytical and the experimental values of beam N-5 (Fig. 7.). In this case the experimental deflections remained constant for almost 1,500,000 cycles and there is a sudden increase at 1,600,000 cycles, whereas the analytical values increase gradually. It should be noted that there is only 5% difference during the last cycles.

Comparison charts for the strengthened beams S-2, S-5, S-6, S-7, S-8, S-9, S-10 and S-11 are shown in Figs. 11, 12, 13, 14, 15, 16, 17 and 18 respectively. These graphs show, that the predicted values are close to the experimental, in almost all of the cases. The difference in all cases is less than 15%. There is only one case where the prediction under-estimates the experimental values. That happened for the beam S-10 (Fig. 17.) and the reason is probably a slippage of the “yoke” on the beam. It should also be noted that the graph for beam S-8 is limited to 20,000 cycles because there was a sudden increase in deflection due to a shear crack. In addition, the maximum applied load was higher than the yielding load. The main reason for using such a high load was to have a big variation of maximum applied loads.

Comparison of strains and stresses at steel and composite is not presented, since the increase in strains was very small in all cases. The analytical model does predict the small increase in strains that happens due to stress redistribution caused by cyclic creep of concrete.

Conclusions

Based on the analysis described, the following conclusions can be drawn:

- The proposed analytical model provides reasonable accurate prediction of deflections, for both reinforced concrete beams and reinforced concrete beams strengthened with composites. Although Glass fiber composites were used for the evaluation, the model is applicable to other types of fibers.
- Cyclic loading lead to increase in deflections for both control and strengthened beams. The deflection increases are slightly lower for strengthened beams.
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Table 1 Summary of monotonic and fatigue test results.

Beam	Testing Frequency	Ultimate Applied Load P _{ult} (kN)	Applied Load at Yield P _y (kN)	Applied Load (kN)		P _{max} /P _{ult}	P _{max} /P _y	Strain on Steel (microstrain)		Strain on Concrete in compression (microstrain)		Number of Cycles to Failure N
				max	min			max	min	max	min	
Non-strengthened reinforced concrete beams												
N-1	monotonic	73.8	58.2	na	na	na	na	2400 ¹	na	3000	na	na
N-2	monotonic	73.3	58.2	na	na	na	na	2460 ¹	na	3100	na	na
N-4	3 Hz	assumed to be the same as N-1 and N-2		31.2	3.3	0.42	0.53	950	60	742	40	2,000,000
N-5	3 Hz			35.6	3.3	0.48	0.61	1100	60	1500	63	2,000,000
N-8	3 Hz			40.0	3.3	0.52	0.68	1493	124	852	71	650,000
N-3	2 Hz			43.6	3.3	0.59	0.74	1701	85	878	44	275,000
N-6	2 Hz			53.4	4.4	0.73	0.91	2010	143	1090	77	155,000
N-7	2 Hz			62.3	3.3	0.85	1.05	2350	99	1500	64	80,000
Reinforced concrete beams strengthened with one ply of GFRP												
S-4	monotonic	109.7	80.1	na	na	na	na	2500 ¹	na	3500	na	na
S-11	3 Hz	assumed to be the same as S-4		40.0	3.3	0.36	0.50	1274	105	950	23	6,000,000
S-10	3 Hz			44.5	3.3	0.41	0.55	1419	106	979	73	685,000
S-2	3 Hz			46.7	2.2	0.43	0.58	1410	66	700	33	880,020
S-5	3 Hz			48.9	4.0	0.45	0.61	1477	101	890	61	800,000
S-7	3 Hz			53.4	3.3	0.49	0.67	1580	99	808	51	570,000
S-9	2 Hz			57.8	3.3	0.53	0.72	1825	110	1090	66	235,000
S-6	2 Hz			64.5	4.4	0.59	0.81	2080	160	1284	99	126,000
S-8	2 Hz			80.1	4.0	0.73	1.0	2350	135	1563	90	30,500

¹ strain in reinforcing steel at general yield, P_y .

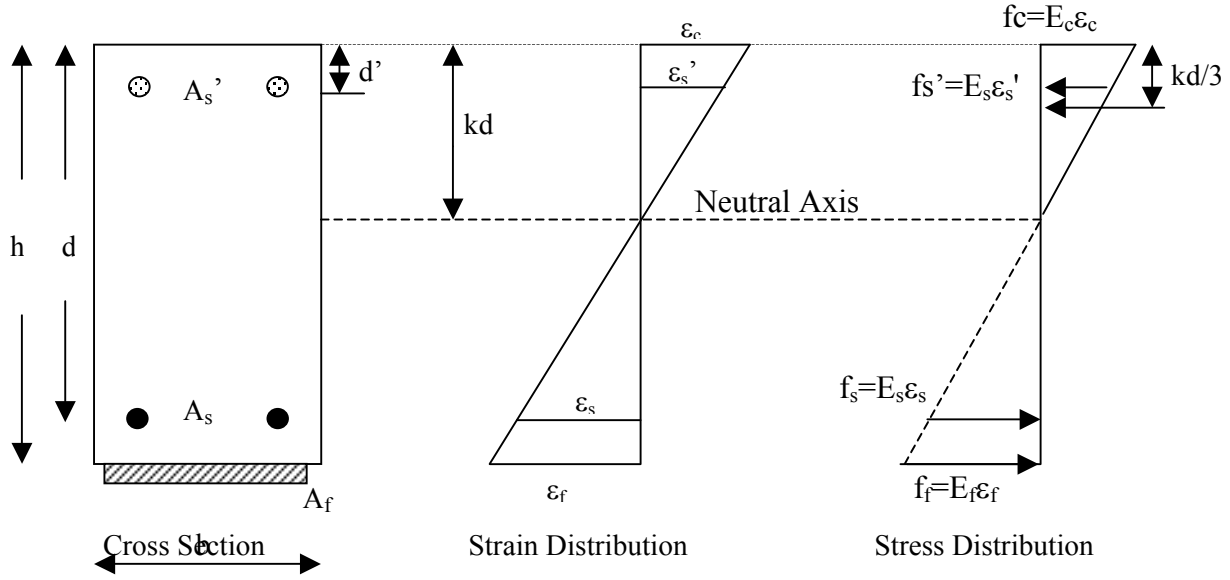


Fig. 1. Strain and stress distributions of strengthened reinforced concrete section at working load

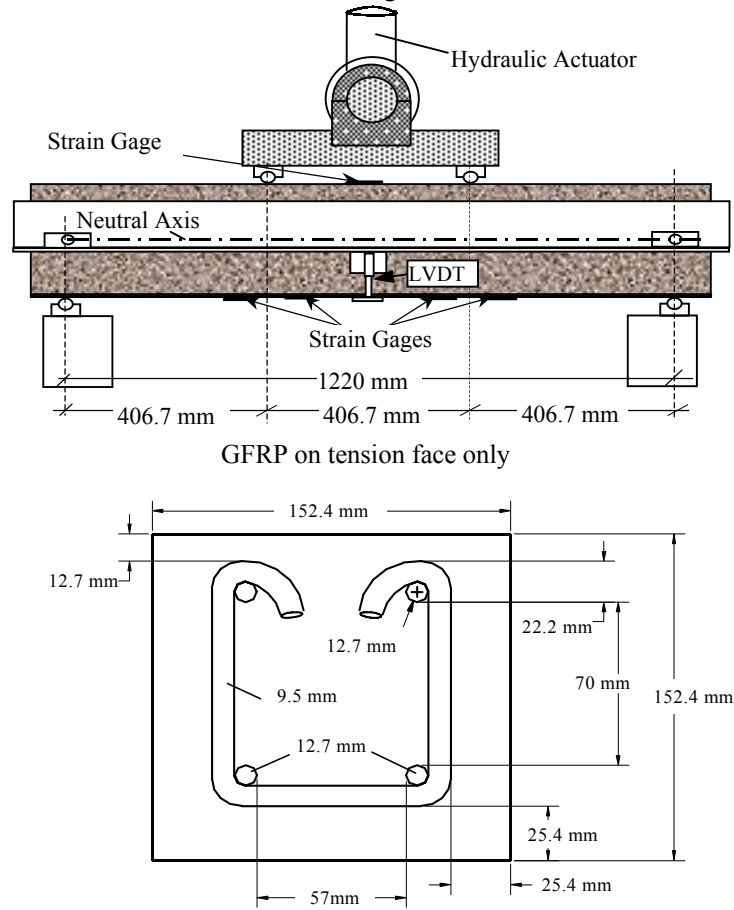


Fig. 2 Test set-up and specimen details.

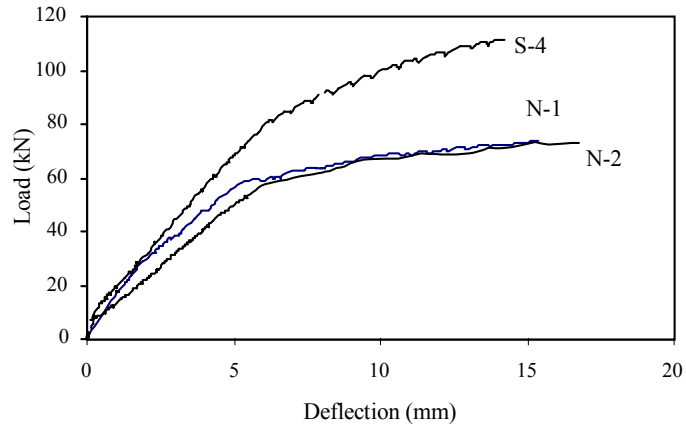


Fig. 3 Load vs Deflection diagram for static tests

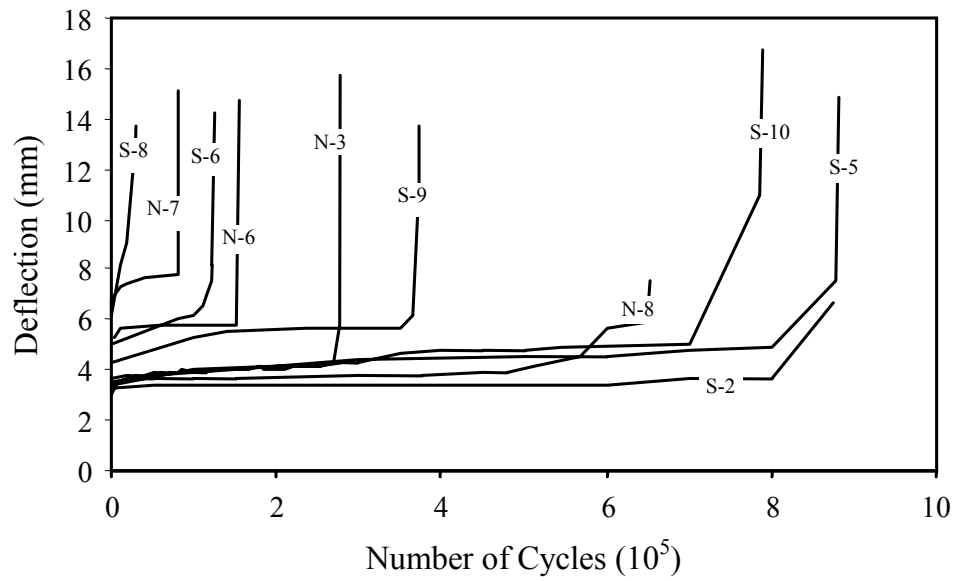


Fig. 4 Deflection vs. number of cycles due to fatigue (5).

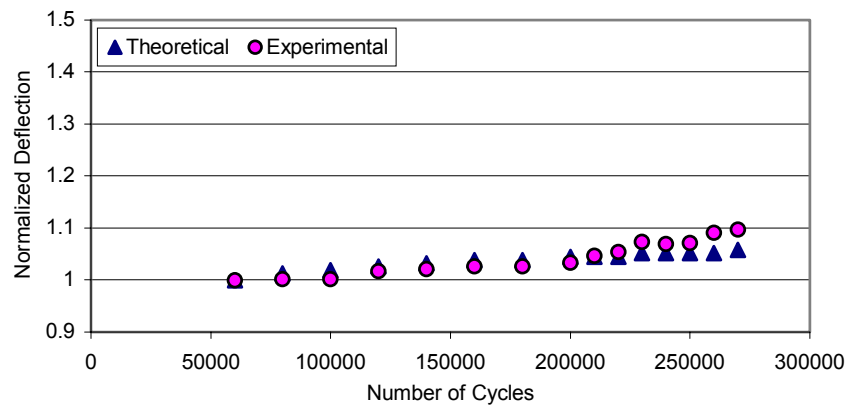


Fig. 5 Experimental and analytical normalized deflection vs. number of cycles (beam N-3)

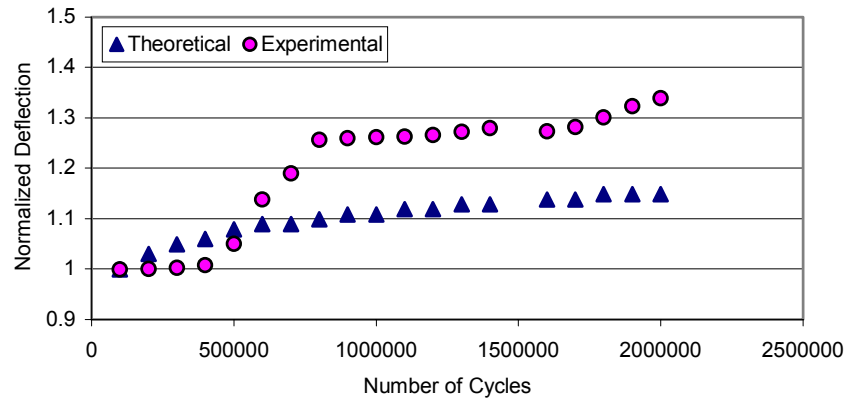


Fig. 6 Experimental and analytical normalized deflection vs. number of cycles (beam N-4)

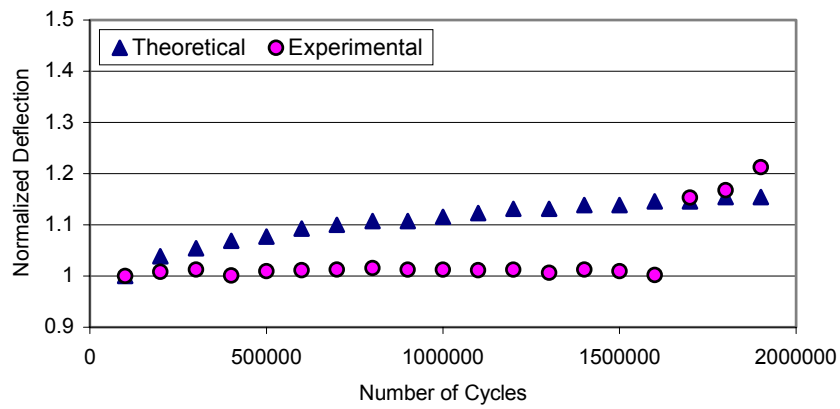


Fig. 7 Experimental and analytical normalized deflection vs. number of cycles (beam N-5)

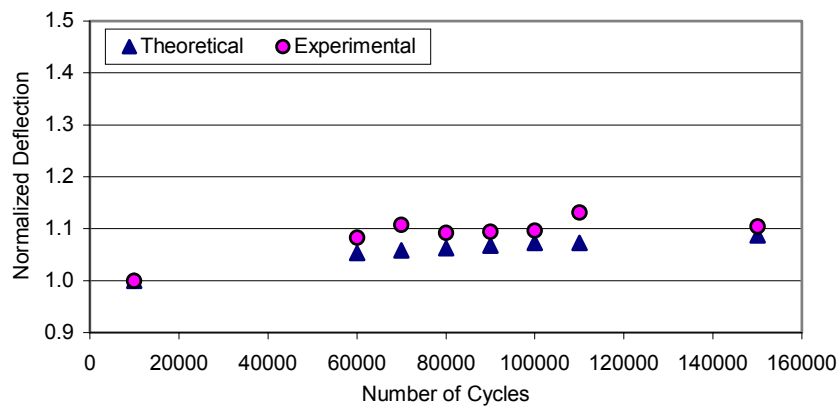


Fig. 8 Experimental and analytical normalized deflection vs. number of cycles (beam N-6)

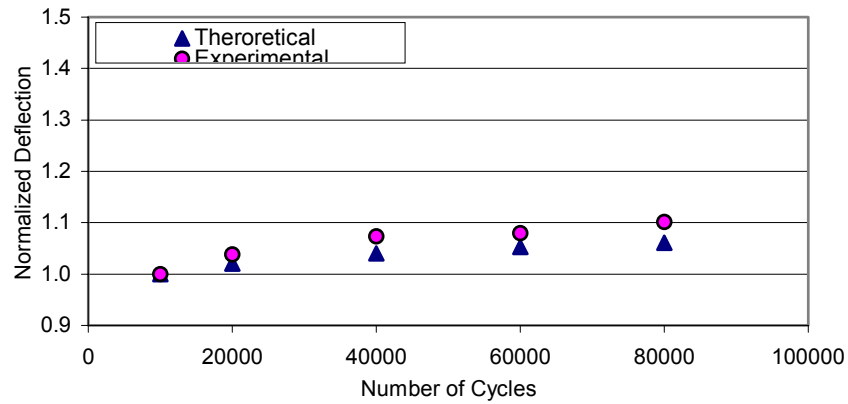


Fig. 9 Experimental and analytical normalized deflection vs. number of cycles (beam N-7)

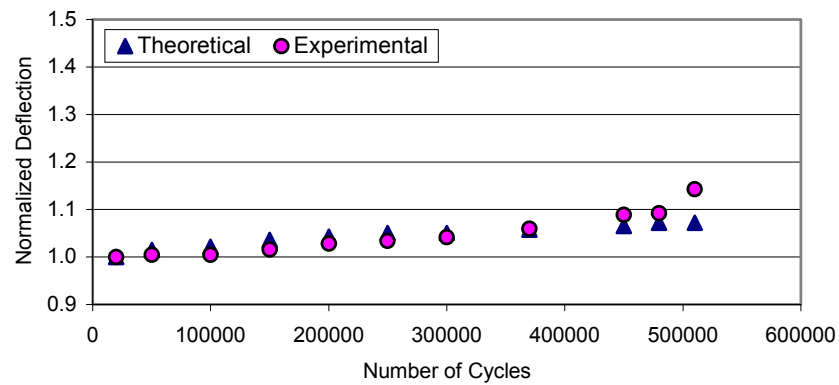


Fig. 10 Experimental and analytical normalized deflection vs. number of cycles (beam N-8)

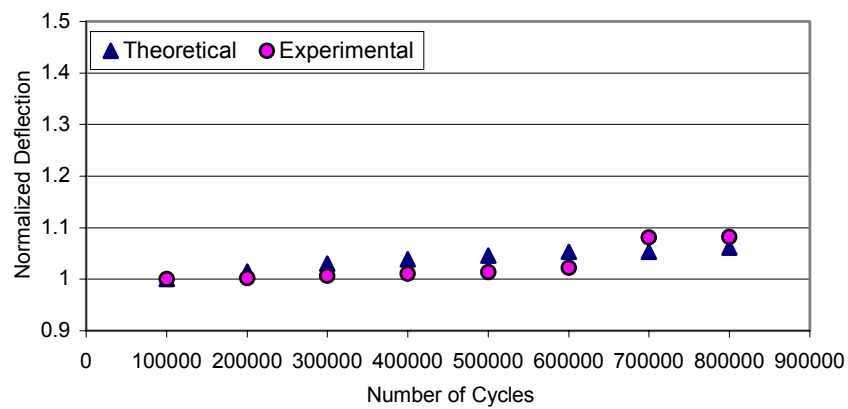


Fig. 11 Experimental and analytical normalized deflection vs. number of cycles (beam S-2)

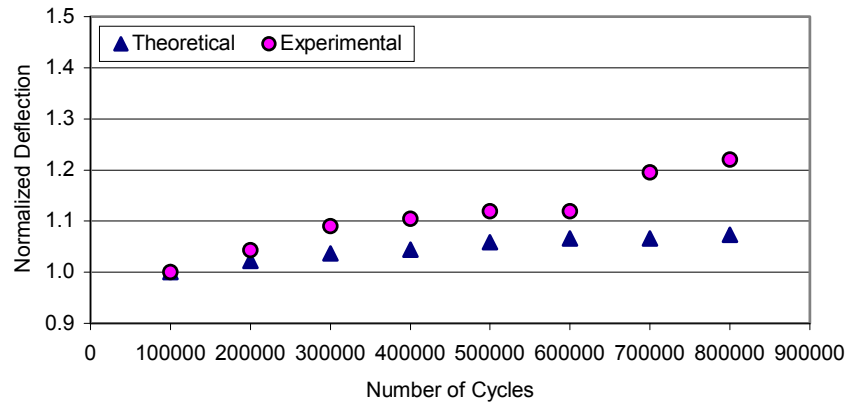


Fig. 12 Experimental and analytical normalized deflection vs. number of cycles (beam S-5)

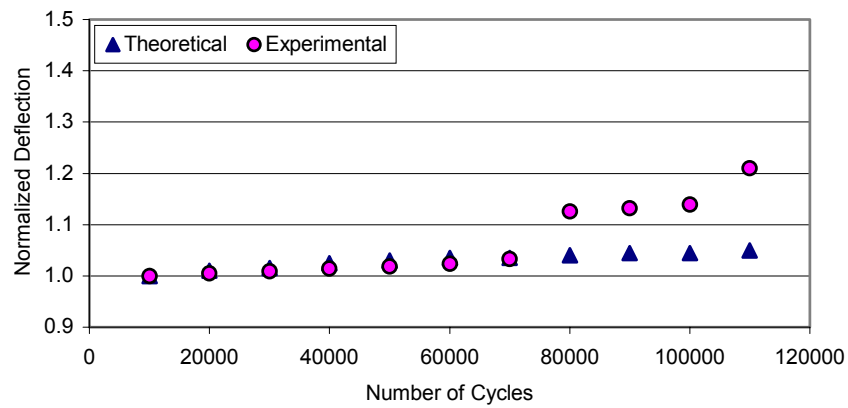


Fig. 13. Experimental and analytical normalized deflection vs. number of cycles (beam S-6)

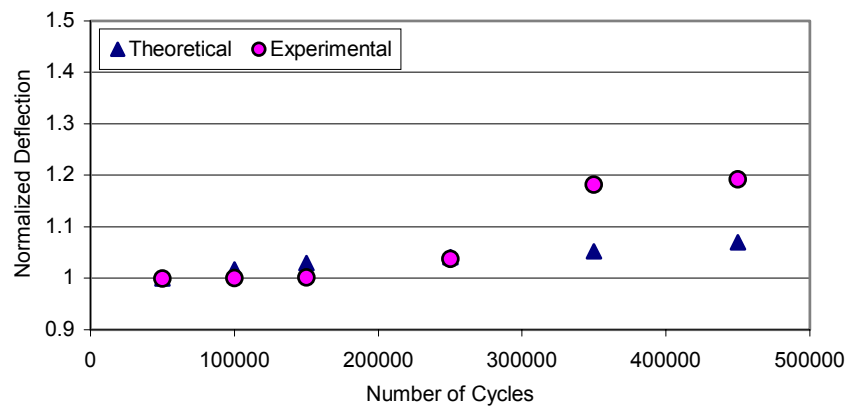


Fig. 14. Experimental and analytical normalized deflection vs. number of cycles (beam S-7)

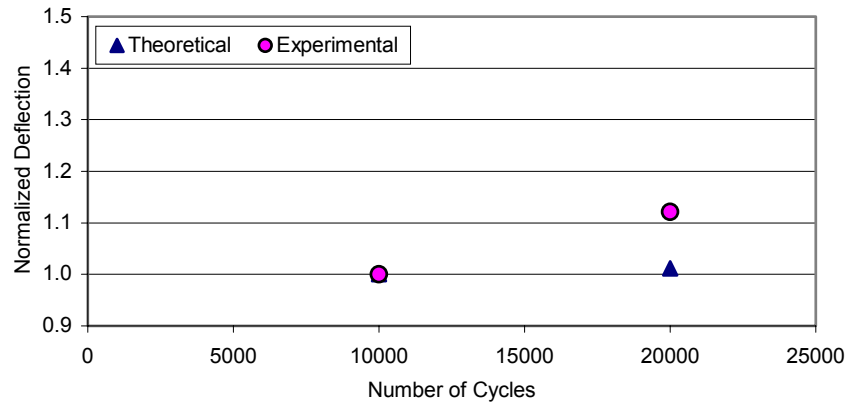


Fig. 15. Experimental and analytical normalized deflection vs. number of cycles (beam S-8)

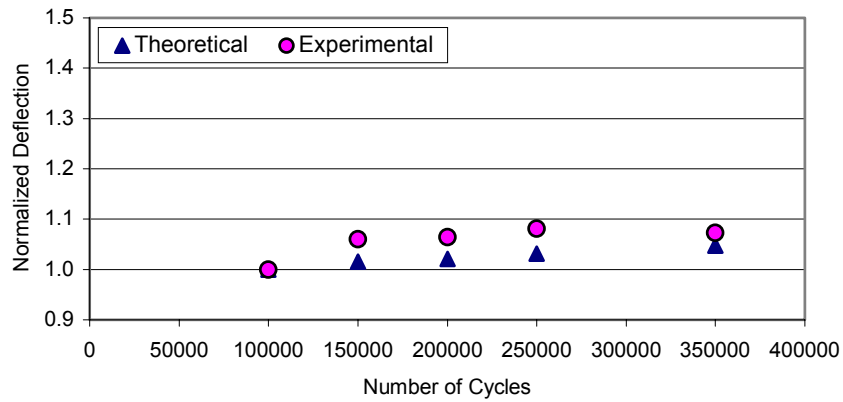


Fig. 16. Experimental and analytical normalized deflection vs. number of cycles (beam S-9)

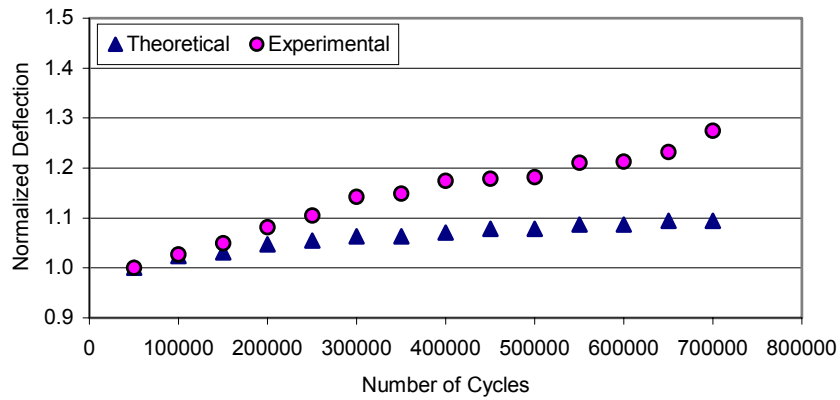


Fig. 17. Experimental and analytical normalized deflection vs. number of cycles (beam S-10)

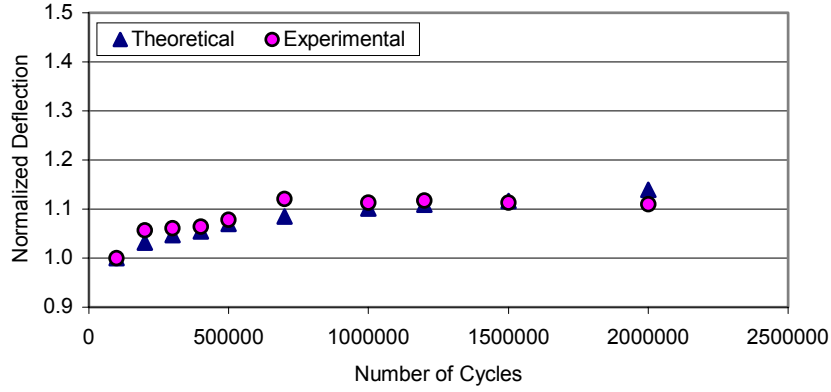


Fig. 18. Experimental and analytical normalized deflection vs. number of cycles (beam S-11)

APENDIX

I. Step-by-Step Procedure for Computation of cycle dependent stresses and deflections.

The procedure described below is for given cross-sectional properties of a reinforced concrete beam strengthened with composite plates, maximum moment, M_a , number of cycles, N , and known properties of the materials.

1. Calculate I_g and \bar{y} from the cross sectional dimensions and properties of the material.
2. Calculate the depth of neutral axis kd , and the maximum compressive stress, σ_{max} , under the static bending moment M_a from the cross sectional properties and the properties of the materials using cracked section analysis
3. Calculate the modulus of rupture of concrete $f_{r,N}$ using equation (13)
4. Calculate $M_{cr,N}$ from equation (17)
5. Calculate the apparent modulus of elasticity of concrete after N cycles of loading, E_N using equation (1) and (2)
6. Calculate the cycle dependent depth of neutral axis, kd_N and $I_{cr,N}$ using E_N and E_s
7. Calculate $I_{e,N}$ using equation (16)
8. Calculate the deflection using equation (15)

II. Numerical Example (Beam S-2 at 800,000 cycles)

Compressive strength of concrete, $f_c' = 40$ MPa, Yield strength of reinforcement, $f_y = 427$ MPa, Modulus of elasticity of concrete, $E_c = 30$ GPa, Modulus of elasticity of steel reinforcement, $E_s = 200$ GPa, Modulus of elasticity of composite, $E_f = 72.4$ MPa, Width of beam, $b = 152$ mm, Height of beam, $h = 152$ mm, Span length, $l = 1220$ mm, depth of the reinforcing bars in the tension side, $d = 109$ mm, depth of the reinforcing bars in the compression side, $d' = 29$ mm, $A_s' = A_s = 258$ mm², $A_f = 129$ mm², Maximum applied load, $P = 46.7$ kN

Maximum moment, $M_a = 9.45$ kNm, Maximum stress on concrete, $\sigma_{max} = 21$ MPa, Minimum stress on concrete, $\sigma_{min} = 1$ MPa, Number of cycles, $N = 800,000$

Gross moment of inertia, $I_g = bh^3/12 = 152 \cdot 152^3 / 12 = 44.5 \times 10^6$ mm⁴

$$\Delta = (\sigma_{\max} - \sigma_{\min}) / f_c' = (21 - 1) / 40 = 0.5$$

$$\sigma_{\max} = (\sigma_{\max} + \sigma_{\min}) / 2 f_c' = (21 + 1) / 2 \cdot 40 = 0.275$$

$$\varepsilon_c = 129 \cdot \sigma_m \cdot t^{1/3} + 17.8 \cdot \sigma_m \cdot \Delta \cdot N^{1/3} = 372.3$$

$$E_N = \frac{\sigma_{\max}}{\frac{\sigma_{\max}}{E} + \varepsilon_c} = \frac{21}{\frac{21}{30000} + 372.3 \times 10^{-6}} = 19584 \text{ MPa}$$

$$f_r = 4 \text{ MPa}, n = \frac{E_s}{E_N} = \frac{200}{19.6} = 10.2, n_f = \frac{E_f}{E_N} = \frac{72.4}{19.6} = 3.69$$

The depth of the neutral axis at 800,000 cycles, k_d can be computed using the force equilibrium equation:

$$b \frac{k_d^2}{2} + (n-1) A_s (k_d - d') = n A_s (d - k_d) + n_f A_f (h - k_d) \Rightarrow k_d = 47.3 \text{ mm}$$

Using the k_d calculated above we can calculate the $I_{cr,N}$:

$$I_{cr,N} = \frac{b k_d^3}{3} + n A_s (d - k_d)^2 + n_c A_c (h - k_d)^2 + (n-1) A_s (k_d - d')^2 = 21.62 \times 10^6 \text{ mm}^4$$

$$f_{r,N} = f_r \cdot \left(1 - \frac{\log_{10} N}{10.954} \right) = 4 \cdot \left(1 - \frac{\log_{10} 800000}{10.954} \right) = 1.8 \text{ MPa}$$

$$M_{cr,N} = \frac{I_g}{k_d} f_{r,N} = 1.69 \text{ kNm}$$

$$I_{e,N} = I_{cr,N} + \left(\frac{M_{cr,N}}{M_a} \right)^3 (I_g - I_{cr,N}) = 21.75 \times 10^6 \text{ mm}^4$$

$$\delta = \frac{23 \cdot M_a \cdot l^2}{216 \cdot E_N \cdot I_{e,N}} = 3.52 \text{ mm}$$

The recorded value for beam S-2 at 800,000 cycles was 3.62 mm