Understanding Teachers' Mathematical Knowledge:

Proportional Reasoning

Form 1

Thank you for agreeing to complete this self-interview.

There are a couple things to remember before you start:

- Please make sure the pen is recording when you start your answer.
- Please remember to think out loud as much as possible.
- Feel free to write anywhere on the sheets of paper EXCEPT the area inside the box. The pen does not record well inside the text boxes.

The next page will give you some guidance and tips to get started. Feel free to doodle all you want on that page – it is practice for you!

If you have any questions, feel free to contact Chandra Orrill (corrill@umassd.edu).

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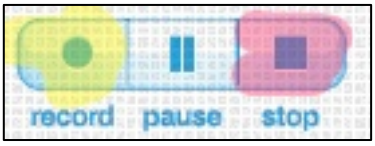
To turn on the pen, use the power button. The pink arrow is point to the power button. When the pen is on, the display (orange arrow) has writing that appears.

When you are Recording the letters “REC” appear in the display.

Be sure there is no pen cap on the pen. See the blue arrow showing what the pen looks like when it is ready to record.

To record, press the Record (highlighted in yellow) “button” printed on the bottom of the page and to stop, press Stop (highlighted in pink). Try it out on this page.



If you want to hear what you recorded, you can click on the playbock tool printed at the bottom of the page. It looks like this:

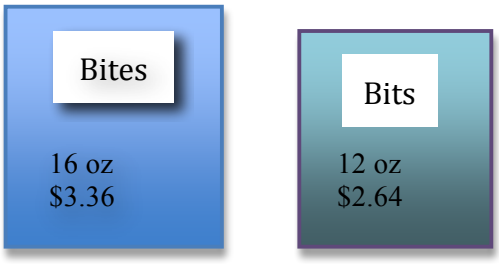
Please try not to fidget or tap the pen. The microphone is very sensitive and extra movements end up creating a lot of noise!

Feel free to “play” on this page. ☺

Bites & Bits

Part 1:

The box of Bites costs $3.36 and the box of Bits costs $2.64.



a) Which is the better buy?
b) How did you decide?

Circle each letter below as you begin your response to its corresponding item above

a b

Bites & Bits
Part 2
The box of Bites costs $3.36 and the box of Bits costs $2.64.

Marc argues that Bits are cheaper because it costs $2.64 while Bites cost $3.36.
Shay argues that Bites is cheaper because it costs $0.21 per ounce while Bits costs $0.22 per ounce.

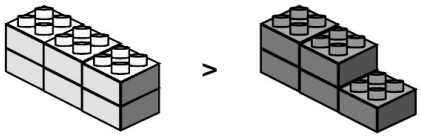
a) How would you characterize what each of the students is thinking?
b) Is one student's approach more appropriate than the other's? Why/why not?
c) How would you help them solve this argument?

Circle each letter below as you begin your response to its corresponding item above

a b c

Blocks Task

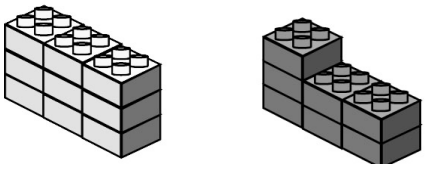
The picture below shows two groups of blocks, 6 light blocks on the left and 5 dark blocks on the right. All light blocks weigh the same amount, and all dark blocks weigh the same amount. The inequality sign indicates that 6 light blocks weigh more than 5 dark blocks.



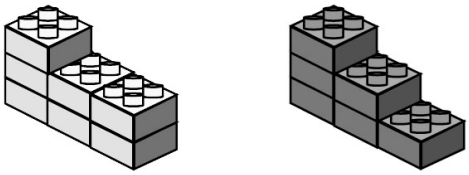
a) Given this, can you determine if the collection of light blocks weighs **more than**, **less than**, or **the same as** the collection of dark blocks for the following cases or if the relationship **cannot be determined**?
b) For which cases can we not determine which collection weighs more?

REMEMBER TO THINK OUT LOUD AS YOU WORK!

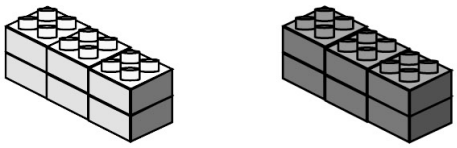
Collection 1



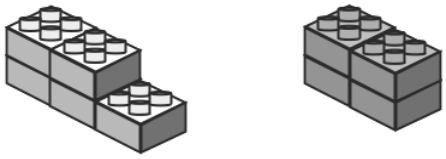
Collection 2



Collection 3



Collection 4

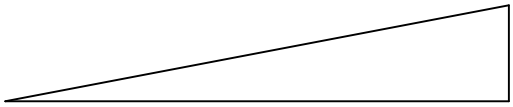


Circle each collection below as you begin your response to its corresponding item above

Collection 1 Collection 2 Collection 3 Collection 4

Triangle

Some students in Mr. Warren's class have noticed that the ratio of 3 feet to 24 feet simplifies to 1 to 8. They also know that this ratio can be written as 1/8. However, they get confused about what the fraction 1/8 means in this situation.



3 ft

24 ft

a) What does the 1/8 mean in this situation?
b) How would you explain that to your students?

Circle each letter below as you begin your response to its corresponding item above

a b

Painting Santa Task

Part 1

Bart is publicity painter. In the last few days, he had to paint Christmas decorations on several store windows. Yesterday, he painted a 56 cm high Santa on the door of a bakery. He needed 6 ml of paint. Now he is asked to make an enlarged version of the same painting on a supermarket window using the same paint. This copy should be 168 cm high. How much paint will Bart need to do this?

Bakery's door **Supermarket window**

Ms. Yarborough gave her students the question above. Please take a moment to work the problem so you're comfortable with it, then we will look at sample responses.

REMEMBER TO THINK OUT LOUD AS YOU WORK!

Painting Santa Part 2:
Ms. Yarborough's students had two favorite answers. About 40% of her class chose 18 ml and about 40% chose 54 ml.

a) What might the students who were wrong been thinking about?
b) Is that something you see commonly with your own students?

Circle each letter below as you begin your response to its corresponding item above

a b

Painting Santa Part 3:
One of Ms. Yarborough's students drew rectangles around the images like this:

Bakery's door

Supermarket window

a) Do you think that is a helpful strategy for a student? Why or why not?

Circle each letter below as you begin your response to its corresponding item above

a

Painting Santa - Part 4:

Bart is publicity painter. In the last few days, he had to paint Christmas decorations on several store windows. Yesterday, he painted a 56 cm high Santa on the door of a bakery. He needed 6 ml of paint. Now he is asked to make an enlarged version of the same painting on a supermarket window using the same paint. This copy should be 168 cm high. How much paint will Bart need to do this?

One of Ms. Yarbrough's students decided to try solving the problem with easier numbers. She thought of the smaller picture as taking 1 tube of red paint and tried to figure out how much red paint would be needed for the large picture. She decided that the right amount would be 9 tubes.

a) Does this method work?
b) Is the students' answer reasonable?
c) Would you want your students to try this approach? Why or why not?

Circle each letter below as you begin your response to its corresponding item above

a b c

Naming Juice Part 1:
A group of fifth graders will mix two types of juice mixture and will put them into jars so they can serve them to the kindergarten students during the week. In the first jar, they put 36 grams of lemon juice and 32 grams of lime juice. In the second jar, they put 20 grams of lemon juice and 15 grams of lime juice. To label the jars, the fifth graders wanted to use one number to accurately represent the lemon-lime flavor in that jar. They considered the following three ideas for labeling the jars:

1) There are 4 more grams of lemon in the first mixture so put 4 on the first label. There are 5 more grams of lemon in the second mixture, so put a 5 on the second label.
2) The fraction of 36/32 for the first type of mixture and the fraction of 20/15 for the second type of mixture.
3) 1.125 (the result of dividing 36 by 32) and 1.33 (the result of dividing 20 by 15).

a) Which idea(s) would accurately indicate the lemon-lime flavor? Why?
b) Which idea do you think is the best for labeling the jars? Why?

Circle each letter below as you begin your response to its corresponding item above

a b

Naming Juice

Part 2:

a) Which of the explanations presented on the previous page could help the fifth graders figure out how much lemon they will need if they use 6kg of lime juice to make a new batch based on the same recipe?

Circle each letter below as you begin your response to its corresponding item above

a

Naming Juice

Part 3:

a) In the third option above (the quantities 1.125 and 1.33), how do these quotients relate to the original recipe?
b) How do 1.125 and 1.33 tell us about the flavor of the mixtures?

Circle each letter below as you begin your response to its corresponding item above

a b

Price Increases

A store raised some of its prices:

- A bottle of dish soap went from $2 to $3
- A box of laundry detergent went from $5 to $6
- A small tube of make up went from $10 to $15
- A large tube of make-up went from $20 to $30

Question 1
The dish soap and the laundry detergent each went up by $1.

a) Does that $1 increase seem equally significant in both cases? (Please explain)

Circle each letter below as you begin your response to its corresponding item above

a

Price Increases

A store raised some of its prices:

- A bottle of dish soap went from $2 to $3
- A box of laundry detergent went from $5 to $6
- A small tube of make up went from $10 to $15
- A large tube of make-up went from $20 to $30

Question 2
The small tube of makeup went up by $5 and the large went up by $10.

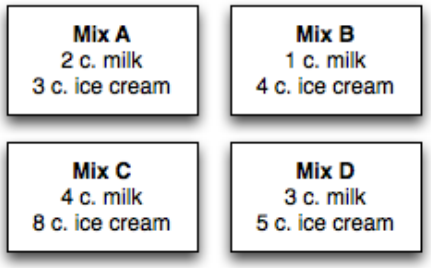
a) Does that mean that the price of the large tube of makeup went up more? Please explain your reasoning.

Circle each letter below as you begin your response to its corresponding item above

a

Milkshakes – Part 1

Ms. Reeves used the following recipes for chocolate milkshakes to help her students learn about proportions. Each recipe includes chocolate ice cream and milk:



a) For each recipe, if the total amount of milkshake changes by scaling the recipe, what stays the same in order to keep the same flavor?
b) For each recipe, what is the constant of proportionality for this situation?
c) For each recipe, is there an easy way to figure out how much milk we would need for 1c of ice cream?

Circle each letter below as you begin your response to its corresponding item above

a b c

Milkshakes - Part 2 – Katrina 1
Ms. Reeves asked her students to consider the following question:

Katrina wants to make Mix A, but she only has ¾ c ice cream. How much milk will she need?

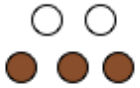
Mix A
2 c. milk
3 c. ice cream

Please solve this task in the space below. Then turn the page to look at student work. (Remember to think out loud as you work!)

Milkshakes - Part 2 – Student Work

Ms. Reeves's students used the following approaches to solve the problem:

Katrina wants to make Mix A (2c milk and 3c ice cream), but she only has ¾ c ice cream. How much milk will she need?



She'll need 2/4 cup. The ratio is 2 parts to 3 parts so it doesn't matter what those parts are as long as they are 2 to 3. In this recipe, it's 2/4 to 3/4.

Student A

a) Describe the approach this student used.

b) Will this approach always work? (please explain why or why not)

Circle each letter below as you begin your response to its corresponding item above

a b

Milkshakes - Part 2 – Student Work (continued)

Ms. Reeves's students used the following approaches to solve the problem:

Katrina wants to make Mix A (2c milk and 3c ice cream), but she only has ¾ c ice cream. How much milk will she need?

Student B

I used a ratio table to figure it out.

milk	2	1	1/3	1/6	3/6
Ice cream	3	1 1/2	1/2	1/4	3/4

Once I did that, it was easy to see that I'd need half a cup.

a) Describe the approach Student B used.
b) Would you want to see your students use an approach like this? (explain)
c) Will this approach always work? (please explain why or why not)

Circle each letter below as you begin your response to its corresponding item above

a b c

Milkshakes - Part 3 – Katrina 2

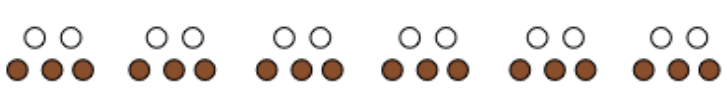
For a few of the students who completed the Katrina question, Ms. Reeves had the following question planned:

Mix A
2 c. milk
3 c. ice cream

Katrina wanted to make 3 c of recipe A. How much of each ingredient did she need?

Work this problem for yourself in the space below. Then, turn the page to look at one student's work. (Remember to think out loud as you work!)

Milkshakes - Part 3 Sample Student Work



30 cups
12 milk
18 ice cream

I drew out one batch then realized that it would be hard to make 3 cups from it. So, I found 30 cups because I could figure out 1/10 of that. Once I figured out there were 12 cups of milk and 18 of ice cream, I just multiplied by 0.10 to find out how much I would need. My answers were 1.2 c milk and 1.8 cups of ice cream.

a) What did this student do?
b) Will this approach work for other problems? All problems?
c) Would you want to see your students use an approach like this? Why or why not?

Circle each letter below as you begin your response to its corresponding item above

a b c

Milkshakes - Part 4: Joshua

Joshua, one of Ms. Reeves' students, was interested in figuring out whether Mix A was more chocolatey than Mix D (the ice cream is chocolate).

Mix A	**Mix D**
2 c. milk	3 c. milk
3 c. ice cream	5 c. ice cream

He decided to divide the amount of milk by the amount of ice cream for each:

$$\frac{2}{3} = .667 \text{ and } \frac{3}{5} = .600$$

a) What does .667 and .600 mean in these answers?
b) What can Joshua do with his result to determine which mixture is more chocolatey?

Circle each letter below as you begin your response to its corresponding item above

a b

Milkshakes - Part 5: Chocolatey

Ms. Reeves asked her students to decide which mix was the most chocolatey and which was the least chocolatey (the ice cream is chocolate).

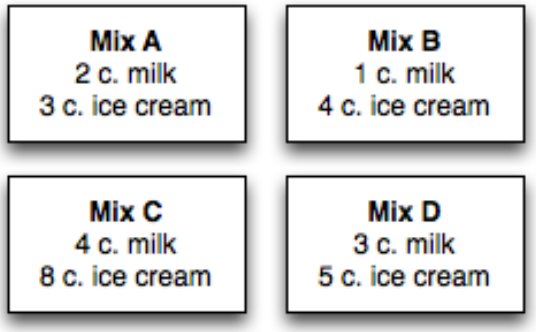
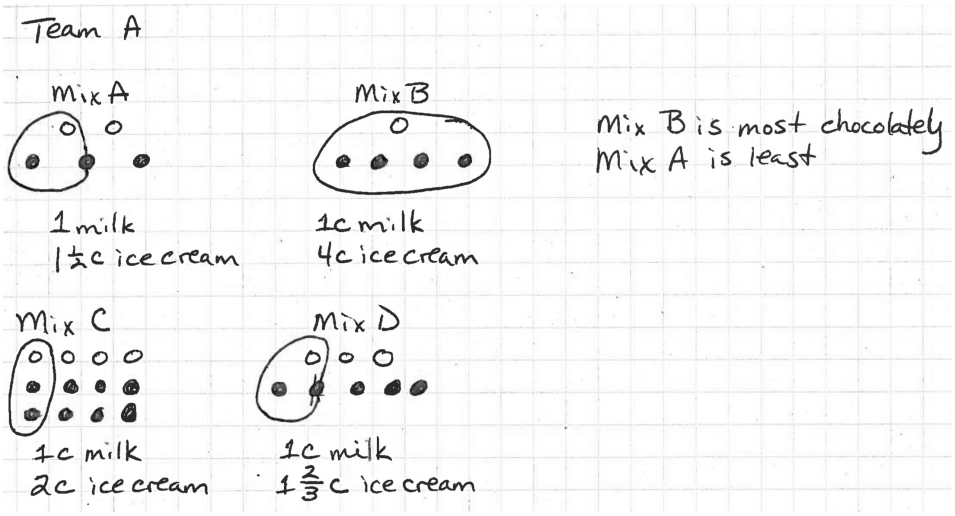


Figure out which mixture is most chocolatey. (Remember to think out loud as you work!)

Milkshakes - Part 5: Student Work

Ms. Reeves often asks her students to use drawings and other kinds of reasoning other than cross multiplication to reason about mathematics. Consider Team A's response:



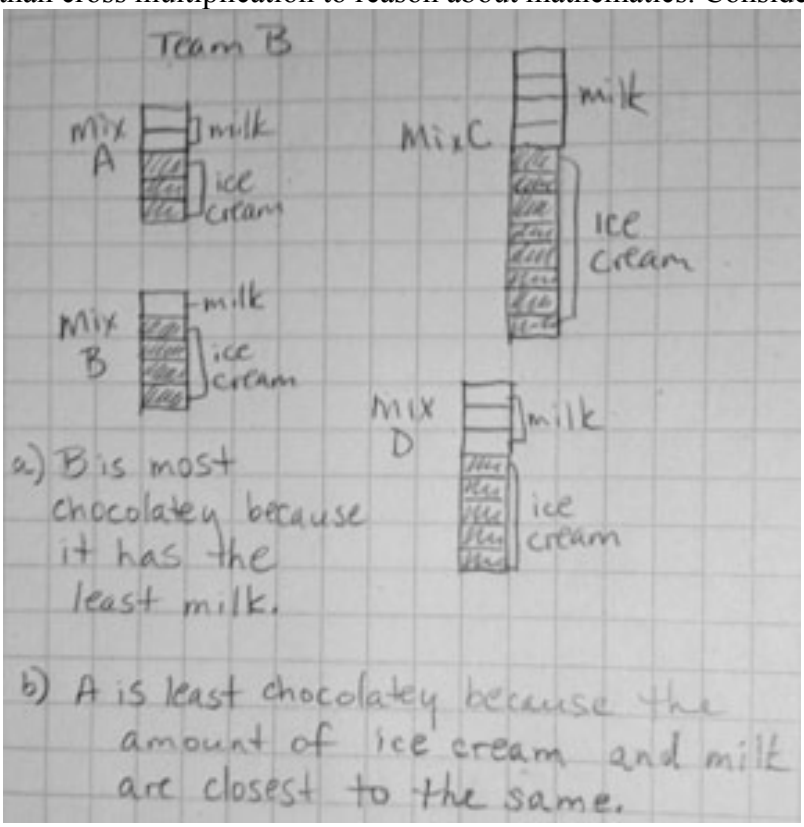
a) How do you describe what the students did?
b) Is this an approach that will always work? Why or why not?
c) Could this approach have worked if they had chosen a number other than 1 for the numerator? Why or why not?
d) What do you think these students understand about the relationship of milk to ice cream in the recipes?

Circle each letter below as you begin your response to its corresponding item above

a b c d

Milkshakes - Part 5: Student Work

Ms. Reeves often asks her students to use drawings and other kinds of reasoning other than cross multiplication to reason about mathematics. Consider Team B's response:



a) How do you describe what the students did?
b) Is this an approach that will always work? Why or why not?
c) Is Team B's drawing useful for answering the question? Explain what is appropriate/inappropriate about the drawing.
d) What do you think these students understand about the relationship of milk to ice cream in the recipes?

Circle each letter below as you begin your response to its corresponding item above

a b c d