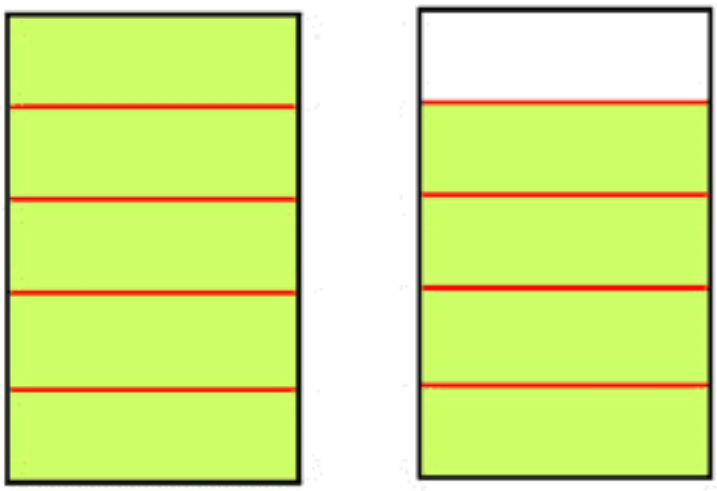
Face-to-Face Interview Protocol from CAREER: Understanding Teachers' Mathematical Knowledge for Teaching (Chandra Orrill – PI)

Beaker Comparison

Prompt:

Samantha and Mike are comparing the amount of liquid in their beaker as shown in the diagram below. Samantha claims that Mike has 20% less than she has. Mike claims that Samantha has 25% more than he has. Who is right?



Samantha **Mike**

Interview Questions

- Who do you think is right? Why?
- How is each person thinking about the amount of liquid in his/her beaker as compared to the amount in the other beaker?

Purpose:

- Appropriateness: can the teacher see both an additive and multiplicative interpretation of a situation?

~~Growing Plants~~

~~Prompt 1~~

~~The growth of 2 plants in the science classroom was measured for one 9-week period. Plant A started out 6 inches long and grew to 9 inches long. Plant B started out 8 inches long and grew to 12 inches long. The science teacher asked the students to compare the plants' growth.~~

~~Take a moment to think about how you would respond to this teacher if you were a student.~~

Growing Plants

Prompt 2

The growth of 2 plants in the science classroom was measured for one 9-week period. Plant A started out 6 inches long and grew to 9 inches long. Plant B started out 8 inches long and grew to 12 inches long. The science teacher asked the students to compare the plants' growth.

What do you think about each student's response?

Sydney:

Plant B grew more because it grew 4 inches, while Plant A only grew 3.

Harrison:

Both grew the same because each is bigger by a third.

Justice:

Start	6	7	8
End	9	10	11

Plant B grew faster because if Plant B grew at the same rate as Plant A, it would have grown to 11 inches, but really it grew to 12, so B had to grow faster.

Interview Questions:

- What do you think of each student's explanation?
    - What is each student doing?
    - What question about the growth of the plants does each student answer?
- What does Justice mean by "rate"? Is that a definition you would normally use?
- How would you rank these students in terms of the sophistication of their responses?

Purpose: This is designed to be a warm-up as it is a pretty typical problem type. It gets at issues of appropriateness because it asks teachers about additive versus multiplicative reasoning. It also asks them to think about different units (e.g., Harrison's explanation).

Thermometers

Prompt: This is a GSP sketch. Participants are given a piece of paper that is blank other than the label "Thermometers" in case they want to write anything as they think about it.

Interview Questions:

Same Questions for both Tabs

A. What happens when you drag the point on the slider??
B. How would you describe the relationship between the two bars as you move the point back and forth?
C. Is there a constant relationship between the bars? (if so, describe that relationship?)
    a. Does that relationship stay the same as you move the slide?
    b. As the top one changes, what happens to the bottom one?
D. Would you characterize this model as showing a proportional relationship? Why or why not?
E. Can you provide a real-world situation or word problem that this model might be showing?
    a. What is proportional in this situation?
F. Can you think of a way to describe the relationship so that I could use the length of one thermometer to predict the length of the other?
    a. Can you provide a mathematical expression or equation that captures this relationship?
G. Is there a scale factor involved in this situation?
    a. If so, what is it?

Follow-Up (if time)

1. Did you approach Tab 2 different from Tab 1?
    a. How was it different?
    b. Why did you use a different approach?
    c. What strategy(-ies) do you think your students might use to figure out the relationship in each tab?

Purposes of item

This is both an appropriateness item and an item that looks at the ways teachers think about proportions visually and dynamically. In this way, it provides data the ends up related to the area tasks. In our pilot interviews, this item helped us understand how the teacher operationalizes the definition for proportion, what relevance a constant relationship has on the proportionality (and what must be constant). We also hear a strong preference between additive and multiplicative language in the ways they describe the situations. While this may or may not align with their knowledge, it seems that the language they are choosing to use may be telling in terms of the level of precision in the language they use in their classrooms.

Potential issues:

- might rely on knowing about equivalent ratios to determine whether this is a proportion. That's okay, though, because it is one of the meanings we want

Critters

For all of these, the participants will be given paper to write things on that will be labeled with "Critters" and the critter with which they are working, but otherwise the prompt is a GSP sketch. I have outlined the questions by Tab name below. (Note: we do not have questions for every tab because we think that would make the interview too long. However, if you think we are missing out on an important opportunity by overlooking a tab, please feel free to comment on that!)

Tab: Kitties 2 Color (We added the color just to make it easier to talk about them – gray kitty and orange kitty).

Interview Questions:

As you begin with kitties, notice that the grey kitty can be moved around if you grab the dot at the bottom of the rectangle.

1. Describe what is happening as you drag the slider. How are the images changing?
2. When we started, the two figures were similar. Where you have ended, are they still similar?
    a. How do you know?
3. How would you characterize the growth as you move the slider?
    a. Is there a relationship between the way the gray kitty grows and the way the orange kitty grows?
    b. Is there anything in the relationship of the kitties to each other that stays the same?
4. Can you describe a rule (in words or symbols) that captures the relationship between the height of the two kitties
    a. When they are not changing
    b. As you adjust the slider
    c. What about the areas of the two kitties?
5. Are there measurable aspects of the grey kitty and the orange kitty that are proportional to each other?
    a. How did you decide?

Tab: Bunnies

1. Describe what is happening as you drag the slider.
    a. How is the image changing?
    b. Are there particular aspects of the picture that are staying the same?
2. When we started, the two figures were similar. Where you have ended, are they still similar?
    a. How do you know?
    b. (If they mention the number of boxes over and up is the same, ask whether it matters that the boxes are not squares)
3. How would you characterize the growth of the bunny as you move the slider?
    a. Is there anything in the relationship of the bunnies to each other that stays the same?
4. How would you compare the sizes of the bunnies?
    a. How would you describe how many times taller the new bunny is than the original bunny?
    b. How would you describe how many times larger the new bunny is than the original?
5. How would you describe the scale factor of the new bunny to the original?
6. Are the boxes in the drawing useful for measuring the bunnies?

Tab: Bears 2

Interview Questions

1. Describe what is happening as you drag the slider. How is the image changing?
2. When we started, the two figures were similar. Where you have ended, are they still similar?
    a. How do you know?
3. How would you characterize the growth as you move the slider?
    a. Is there anything in the relationship of the bears to each other that stays the same?
4. How would you describe how many times larger the new bear is than the original?
5. How would you describe the scale factor of the new bear to the original?
6. If you drew a line from the bear's big eye to his hand, how would it change as you drag the slider?
    a. Would the relationship of the length of the line to the height of the bear stay the same or would it change as the bear is resized?
    b. How would it relate to a line drawn between the same eye and hand of the original?
7. What is the ratio of the bear's belly to the frame?
    a. Does that ratio stay the same as the bear changes size?
8. Can you provide a mathematical rule that will allow someone to use the position on the slider to:
    a. Predict the width of the new Bear?
    b. Predict the perimeter of the new Bear?
    c. Predict the area of the new Bear?
    ~~d. Given what you've seen with this bear, how many times larger would the new bear be than the original if we could drag the slider to 7? (why?)~~
9. Does the _____ of the Bear vary proportionally with the value of the slider as the size changes?
    a. Area
    b. Perimeter
    c. Height (or Width)
10. Sometimes people describe the proportional relationship by defining a constant. For example, a certain trail mix may have 2 times the amount of peanuts as M&Ms. Thus, one could figure out the amount of peanuts by multiplying the amount of M&Ms by two.
    a. Is there anything in the Bear situation that is constant like that?
    b. Is there something in the relationships on the screen that stays constant to let us predict the size of the new Bear?
    c. Could you express this as a mathematical expression or equation?

Purpose

This item is designed to pursue the emerging findings we have related to area and proportion as well as the difference in ways the teachers think about dynamic relationships versus static ones. The questions help us understand how the teachers think about various aspects of proportionality (e.g., constant relationship, relationship between "growth" and proportion).

Oil & Vinegar Task

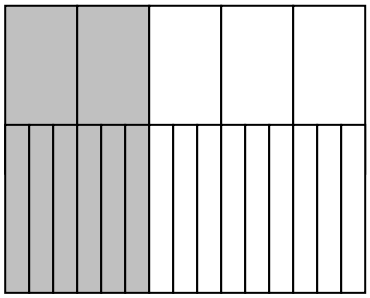
Prompt 1:

The teachers were discussing the following item in a professional development course:

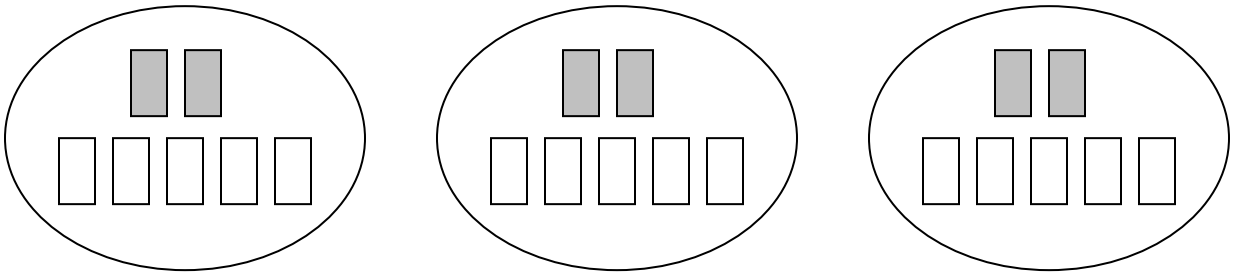
> *Alexi made a batch of salad dressing using 2 tablespoons of vinegar and 5 tablespoons of oil. She would like to make a much larger batch that preserves the ratio of vinegar to oil. If she uses 15 tablespoons of oil, how much vinegar should she use?*

One of the teachers said:

"I'm confused about this problem. If I think about $\frac{2}{5}$ *and* $\frac{6}{15}$ *as equivalent fractions, then the amounts represented by both are the same and* $\frac{3}{3} = 1$.



But if I think about 2:5 as a ratio and triple the batch of salad dressing, I get a lot more salad dressing, and I think I'm multiplying by 3 not $\frac{3}{3}$.



How would you respond to this teacher?

Interview Questions

1. Is the teacher multiplying by 3 or 1?
2. Would you use either representation in your class? Why/why not?

Purpose: Fraction/Ratio connection

Oil & Vinegar Task

Prompt 2:

> *Alexi made a batch of salad dressing using 2 tablespoons of vinegar and 5 tablespoons of oil. She would like to make a much larger batch that preserves the ratio of vinegar to oil. If she uses 15 tablespoons of oil, how much vinegar should she use?*

Another teacher in the professional development, Mr. Miller, said that he would explain situations like the Alexi task by setting up the proportion:

$$\frac{2}{5} = \frac{x}{15}$$

Then, he would explain that the answer is 6 because $\frac{2}{5}$ and $\frac{6}{15}$ are equivalent fractions and $\frac{2}{5} \bullet \frac{3}{3} = \frac{6}{15}$.

What do you think of Mr. Miller's explanation?

Interview Questions

3. What do you think of Mr. Miller's explanation?
4. What does 2:5 mean as two-fifths? What is there 2/5 of in this situation?
5. How would you react to the follow ideas from other teachers about what the 2/5 means as a fraction?

Oil & Vinegar Task

Prompt 3:

> *Alexi made a batch of salad dressing using 2 tablespoons of vinegar and 5 tablespoons of oil. She would like to make a much larger batch that preserves the ratio of vinegar to oil. If she uses 15 tablespoons of oil, how much vinegar should she use?*

In the professional development, some teachers reacted to Mr. Miller's suggestion by adding the following comments. Please react to each of these comments about what the 2/5 means:

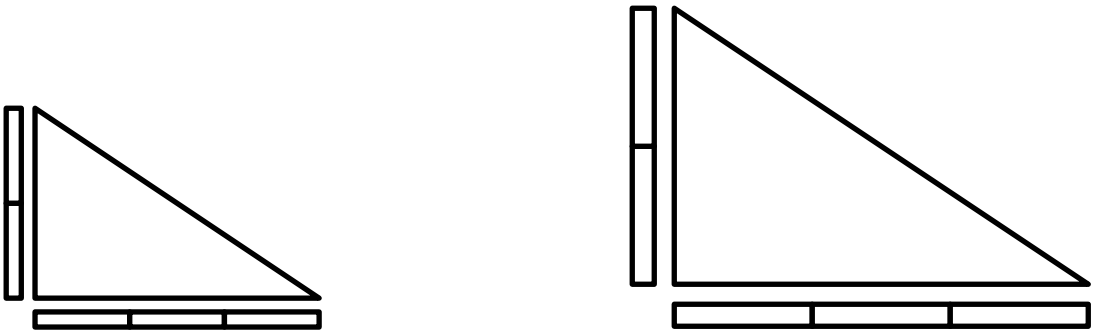
- "I know that 6 is $\frac{2}{5}$ of 15. So I guess there's a two-fifth there."

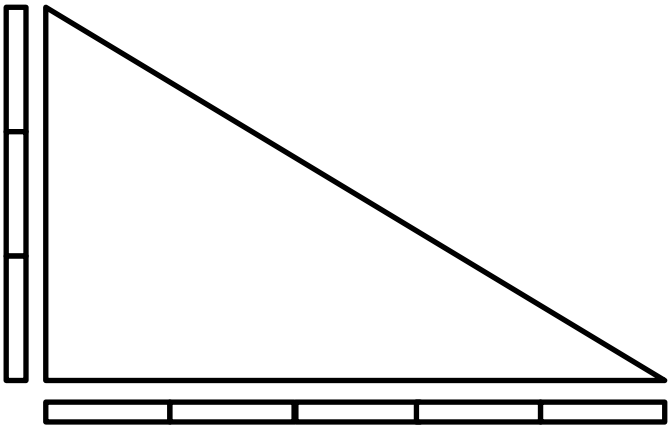
- "Fractions and ratios are the same thing."

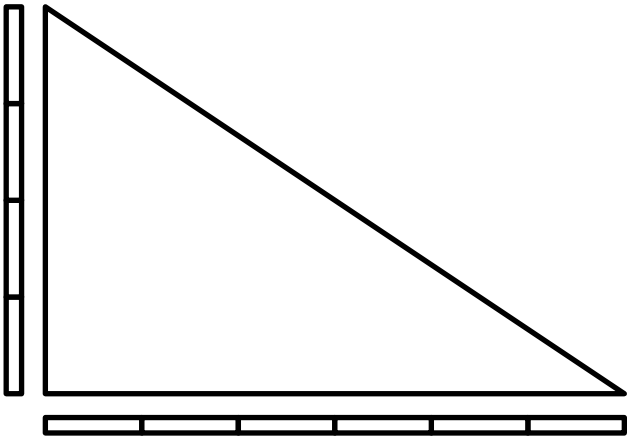
- "$\frac{2}{5}$ is a ratio here not a fraction. A fraction is a part-whole relationship like 2 T vinegar to 7 T of salad dressing, which is $\frac{2}{7}$ not $\frac{2}{5}$."

- "I wonder if it has something to do with finding how much vinegar I would need for 1 part oil or how much oil for 1 part vinegar?"

Similar Triangles
Would you consider any of the triangles below similar?
What attributes are you considering when making your decision?







Purpose
Looking at Fixed parts of Variable Size.

Interview Questions

1. How do you define proportional relationships for your students?
    a. How do you think about proportional relationships for yourself?
    b. How did you learn to do proportions?
    c. How do you think about the relationship between a ratio and a proportion?
2. To solve a proportion, we often set two ratios equal to each other. So, in the case of the salad dressing we were talking about, we might say: 2/5 = x/15.
    a. What does it mean for these to be equal?
3. Do you teach about constant of proportionality?
    a. How do you define it for students?
    b. Is that the same way you think about it for yourself?
    c. What is it showing?
    ~~d. How can you use it?~~
4. When you think about ratios and fractions, do you see them as being the same or are there differences?
    ~~a. Is "equivalence" the same for both?~~
    b. If we think about a free throw context, we can say that a player made 2 of 3 thrown in the first half of the game and 3 out of the 4 thrown in the second half of the game and treat it as: $\frac{2}{3} + \frac{3}{4} = \frac{5}{7}$. But, if we add the fractions 2/3 and ¾, the answer is 17/12: $\frac{2}{3} + \frac{3}{4} = \frac{17}{12}$. Why do fractions and ratios work differently? Does the free throw way of adding always work for ratios?
        i. Once we add the results of the two halves together, is that result (5/7) a ratio, a fraction, or something else? (why is it/is it not a ratio? Why is it/is it not fraction?)
5. When you think about similarity, do you think about it being related to proportional relationships?
    a. When you teach this, do you talk about proportions and similarity together at all?
6. Do you teach scale factor?
    a. How do you define it for students?
    b. Is that the same way you think about it for yourself?
    c. What is it showing?
    d. How can you use it?
7. How do you think about unit rates?
    a. If a batch of salad dressing has a 2 to 5 ratio of vinegar to oil, what is the unit rate.

~~8. A restaurant sells large and small pizzas. Each pizza is cut into 6 pieces.~~
    ~~a. How much pizza are you eating if you eat one piece of the large pizza? What about one piece of the small pizza?~~
    ~~b. Without knowing the exact sizes of the pizzas, could you compare the amount of pizza in one slice of the small to one slice of the large?~~
    ~~c. Without knowing the exact sizes of the pizzas, could you compare the ratio of the area of one piece of the small to the area of one piece of the large?~~

9. Thinking back to the salad dressing problem one last time, if a student describes the proportion of vinegar to oil as 2/5, would you accept that response? Why or why not?
10. Thinking back to the interview we mailed you, there was a question about painting Santa on a window. In that task, you were asked some questions about the amount of paint you would need to paint a larger Santa versus a smaller Santa.
    For this situation, consider that you made a deal with 2 friends to share the cost of the paint. You figured out together that for the smaller Santa the amount of red, white, and black paint needed were the same, so each of you purchased one color of paint. If you decided to paint a Santa that is three times as wide, would it still be fair to share the paint cost this way?
    - If they ask about 3 times as long as well as 3 times as wide, ask 'does it matter'?
11. Why did you decide to become a mathematics teacher?
12. How did you learn about proportions?
13. Did you go through a traditional teacher preparation program?
    a. If yes, undergraduate or graduate?
    b. If no, how did you get certified?
14. For any term they used (e.g., "rate"):
    a. How do you define xxxxx for your students?
    b. How would you tell your students that xxxxx relates to proportions?
    c. How might your students use xxxx to think about a problem in your class dealing with proportions?