UMass | Dartmouth
UNIVERSITY STUDIES

Master Syllabus - *Annotated Template*
Course: *MTH461 Small-World Networks*
Cluster Requirement: *5B*

Course Overview:

Modeling small-world networks. This experiential-learning course focuses on the simulation and analysis of small-world networks, including social networks, food chains and the world-wide web. Models will include regular lattices, random graphs, Strogatz-Watts networks, and random accretion models of Barabási and Albert and of Aeillo, Chung and Lu.

University Studies Course Rationale:

Networks are an increasingly active area of research in mathematics. This course takes advantage of the availability of computing power and mathematical software to allow students to explore fundamental aspects of small-world networks through simulation

Readings from the popular science literature, and to a limited extent, in-class lectures will introduce specific claims about the nature of real networks, introduce the small-world property, and describe models that might have this property and be candidates to model the real networks. In discussion based on these readings, students will articulate how small-world networks describe real networks relevant to communities of which they are members. These networks may be part of the community itself, or the subject of study of the student's professional community.

Students will work in small groups on a sequence of projects, each focused on a particular network model such as a regular lattices, random graphs, and the Strogatz-Watts model. They will create simulated networks and calculate their properties using Mathematica or an equivalent mathematical software tool. For each of these projects, they will write a short report, including answers to specific questions about the model and about mathematical aspects of their findings.

As the semester progresses, students will reach the point where they can develop their own models for specific networks they will find in their larger community. They will work on these as final projects, which they will present to their peers and publish online. In the process of developing topics they will discover and articulate needs of their own communities. In the research for the project, they will use what they have learned in the course of the semester to address such a need, or describe how it has been addressed by the work of others, by applying small-world network models. In their paper, they will describe the connection between what they have learned and the specific issue this can address. They will demonstrate the value of this engagement in their presentations and online publications.

Learning Outcomes:

<u>Course-Specific Learning Outcomes:</u>

Students will be able to construct computer models of the specific types of networks listed above, as well as others typically used to model real small-world networks. They will be able to identify and compute measures of the small-world property, interconnectedness and clustering. They will be able to prove propositions about each of these types of networks. Having gained this understanding, they will seek out exemplars in the larger world and apply the techniques they have learned to model a real small-world network in a field of interest to them. These networks might include predator-prey networks, neuronal networks, protein relatedness networks, friendship networks, terrorist networks, local

subnetworks of the internet, local subnetworks of the world-wide web, epidemiological models or models of economic interactions, just to list some easily-found examples. They will communicate their findings in clear mathematical terms.

University Studies Learning Outcomes: *This second part lists the relevant learning outcomes that this course meets as described in the University Studies Cluster requirement. This part should list the Cluster and letter of the Cluster requirement (e.g., Cluster 2A) and should simply reproduce verbatim those learning outcomes here. Additionally, if the Cluster requirement provides alternative outcomes, the specific option to be applied in this course should be identified. For example, in Cluster 4A, courses can fulfill the requirement by explaining different perspectives on: a) what it means to be human and how the significance of human existence has been understood; b) the nature of human relationships and how these relationships are evidenced in regard to the broader world; or c) how knowledge is obtained, maintained and changed, as well as how individuals come to understand and think about the world around them. The outcomes here should identify which option this course intends to use to fulfill this requirement.*

5B. Learning Through Engagement

Upon completion of this requirement, students will be able to:
1. Identify the needs and resources of the communities to which they belong.
2. Apply knowledge and skills gained through academic study to real problems and/or opportunities within their communities.
3. Describe the connections between learning on campus and the issues and needs of broader academic, professional or civic communities.
4. Articulate the value of engagement to other members of their communities.

In this course, students will choose the community to which they will connect their learning as part of their choice of term project. They will be engaging in independent research in narrowing their focus to a particular network. In this phase they will have to identify a need that can be addressed by means of network modeling. They will apply skills learned in this course to construct a mathematical model to capture aspects of a network they have identified as important to the community they have chosen. In their paper and in public postings on one or more websites they will connect the mathematics learned in the classroom with some aspect of this community. Their final product will demonstrate the value of engagement to other members of their communities.

Examples of Texts and/or Assigned Readings:

Nexus: Small Worlds & the Groundbreaking Science of Networks; M. Buchanan; Norton
Mathematica (mathematical programming software)
The Probabilistic Method; Noga Alom & Joel H. Spencer; Wiley
Sync: How Order Emerges From Chaos In the Universe, Nature, and Daily Life; Steven H. Strogatz; Hachette

Example Learning Activities and Assignments:

1. Students will read *Nexus* in periodic installments. These include chapters describing aspects of friendship networks, relationship networks, authorship networks, and collaboration networks, among others. The students will find many of these apply to their own communities. They will describe these findings in class, as part of the discussion of the readings.
2. The course culminates in a **project** for which students must present an application of the mathematics of small-world networks in some larger context. They may choose between collecting data and constructing the model themselves, or giving a concise description of a model already in the literature.
    a. To come up with a topic for the project, students will either be looking for problems to address from within their personal communities, or exploring problems already documented in some larger, most likely professional, community. Students will document, as part of their **topic proposal**, what resources they have found (which might blogs, journals, personal conversations,

etc.) to describe networks within, or relevant to, this community that may be modeled in this way.

b. The project itself will be a **term paper** describing the network itself, the mathematical model, and relevant findings from the model.
c. Students will **publish their findings online**. This will either be on a specialized blog pertaining to the community whose problem is addressed by the model, or in a blog specific to this course.
d. Students will **present** their work in a forum open to other students in Mathematics, either as short talks or as posters.

Outcome Map:

Univ St Learning Outcome	Teaching and Learning Activities	Student Work Products
1 5B 1	Readings in Nexus, topic proposal, term paper (introductory section)	Topic proposal, Term paper (introductory section)
2 5B 2	Research/modeling required for term paper	Term paper
3 5B 3	Writing term paper	Term paper (introductory & concluding sections)
4 5B 4	Writing to publish online, presentations	Online publication; poster or talk
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Sample Course Outline:

MTH461 Small-World Networks

Course Description
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Learning Outcomes:

The successful student will demonstrate:

- Appropriate use of mathematical software to model small-world networks of the specific types detailed below.
- Use of standard measures to describe properties of such models.
- Calculation of these measures for regular networks.
- Prediction of measurements of these properties for specific forms of randomly-generated networks, and tests of these predictions.
- Familiarity with main theoretical results of Erdös & Rényi.
- Ability to apply the above to model and/or analyze specific networks, or types of networks, found directly or in literature from other disciplines.

In short, the student will be able to read a claim of the sort Buchanan cites about a given type of network, use software to generate model networks of that type, and check whether the claim holds true for those networks.

Textbook:

Nexus: Small Worlds & the Groundbreaking Science of Networks M. Buchanan
Mathematica or equivalent mathematics-programming software

Workload

There will be 6 homework assigments, a midterm, a final paper, and a presentation to the class. The typical homework assignment asks you to continue work on software projects begun in class, answer questions based on those models, and answer analytic questions suggested by their discoveries. Some of the latter will be of the form: "State and mathematically prove a conjecture based on what you have observed."

Graduate students taking this project-based class as MTH561 will be expected to show greater sophistication in their projects (in mathematical content and/or in quality of programming, depending on their field of study), to answer more theoretical questions on topics such as Erdös-Rényi random graph theory, and to address questions in their final papers that have the potential to develop into publishable original work.

Topic syllabus

Week I:
Introduction
Small worlds & networks

Week II:
Graphs & their computer representations
Density, diameter & clustering

Week III:
Random graphs (Erdös & Rényi)

Week IV:
Generation & properties of random graphs

Week V:
Mathematical certainties about random graphs

Week VI:
More on random graphs

Week VII:
Social networks & and power companies
Granovetter, Strogatz & Watts

Week VIII:
Generation & properties of Strogatz-Watts networks
Midterm exam

Week IX:
The Internet, the web
Power law distributions of vertex degrees

Week X:
Generation & properties of Barabási-Albert networks

Week XI:
Aiello-Chung-Lu models
Mathematical results

Week XII:
And more mathematical results

Week XIII:
Paper draft due
Related topics

Week XIV:
Student presentations